

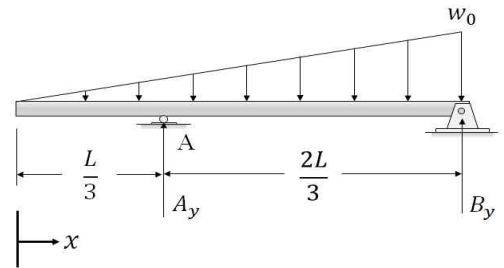
2016년도 구조역학 특론 시험 안내

구조역학 특론의 시험문제를 아래의 문제 중에서 출제할 예정이며 매우 엄격히 평가할 예정이므로 반드시 풀어보고 응시할 것.

담당교수

1.

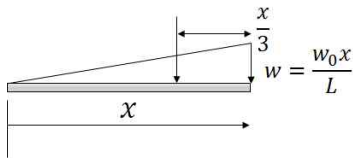
1) Determine the support reactions of the beam shown in Fig.



ANS : $A_y = \frac{w_0 L}{4}$ $B_y = \frac{w_0 L}{4}$

2) Determine the shear(V) and moment(M) equations throughout the beam as functions of x.

$0 < x < \frac{L}{3}$:



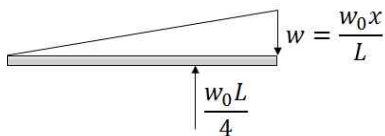
$$V = -\frac{1}{2} \frac{w_0 x}{L} \cdot x = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^2}{2L} \cdot \frac{x}{3} = -\frac{w_0 x^3}{6L}$$

ANS : $V = -\frac{w_0 x^2}{2L}$

ANS : $M = -\frac{w_0 x^3}{6L}$

$\frac{L}{3} < x < L$:



ANS : $V = \frac{w_0 L}{4} - \frac{w_0 x^2}{2L}$

ANS : $M = \frac{w_0 L}{4} \left(x - \frac{L}{3}\right) - \frac{w_0 x^3}{6L}$

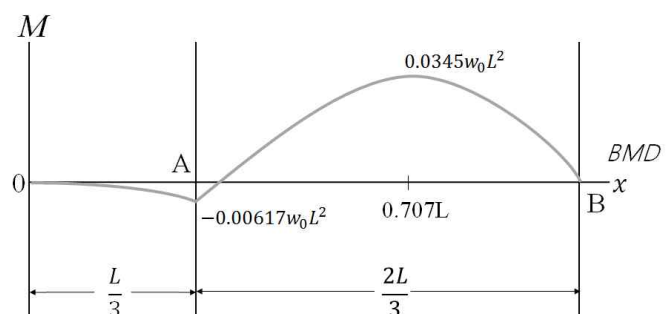
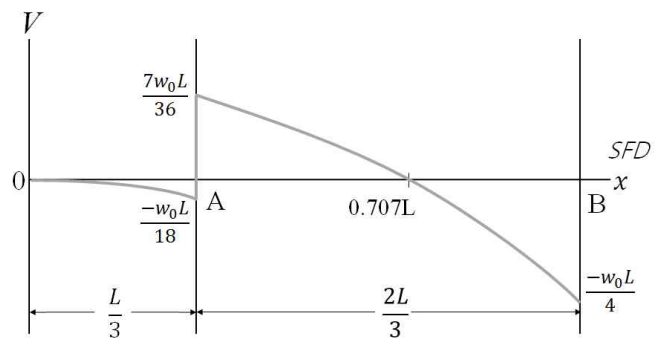
3) Draw the shear and moment diagram of the beam. Indicate the absolute maximum shear force (V_{\max}) and the maximum bending moment (M_{\max}) at the diagram.

$$V_{x=\frac{L}{3}} = -\frac{w_0}{2L} \left(\frac{L^2}{9}\right) = -\frac{w_0 L}{18}$$

$$V_{x=\frac{L}{3}} = -\frac{w_0 L}{18} + \frac{w_0 L}{4} = \frac{7w_0 L}{36}$$

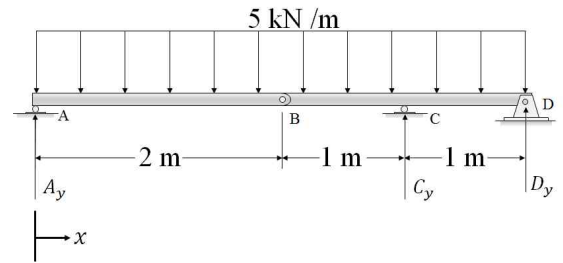
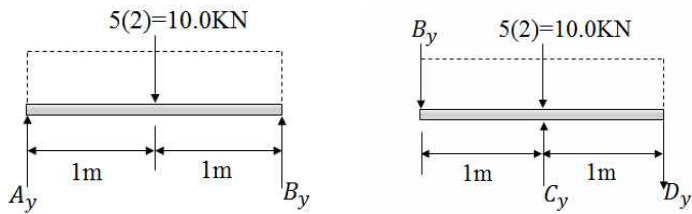
$$V = \frac{w_0 L}{4} - \frac{w_0 x^2}{2L} = 0 \quad \therefore x = 0.707L$$

$$M_{\max} = \frac{w_0 L}{4} \left(0.707L - \frac{L}{3}\right) - \frac{w_0}{6L} (0.707L)^3 = 0.0345w_0 L^2$$



2.

- 1) Determine the support reactions of the beam shown in Fig.



$$\curvearrowleft + \sum M_A = 0; \quad B_y(2) - 10.0(1) = 0 \quad B_y = 5.00 \text{ kN}$$

$$+ \uparrow \sum F_y = 0; \quad A_y - 10.0 + 5.00 = 0 \quad A_y = 5.00 \text{ kN}$$

$$\curvearrowleft + \sum M_C = 0; \quad 5.00(1) + 10.0(0) - D_y(1) = 0 \quad D_y = 5.00 \text{ kN}$$

$$+ \uparrow \sum F_y = 0; \quad C_y - 5.00 - 5.00 - 10.0 = 0 \quad C_y = 20.0 \text{ kN}$$

$$\text{ANS : } A_y = 5 \text{ kN} \quad C_y = 20 \text{ kN} \quad D_y = 5 \text{ kN}$$

- 2) Determine the shear (V) and moment (M) equations throughout the beam as functions of x.

$$0 < x < 3 \text{ m} :$$

$$V = 5 - 5x$$

$$M = 5x - 5x \cdot \frac{x}{2}$$

$$\text{ANS : } V = -5x + 5$$

$$\text{ANS : } M = -\frac{5}{2}x^2 + 5x$$

$$3 \text{ m} < x < 4 \text{ m} :$$

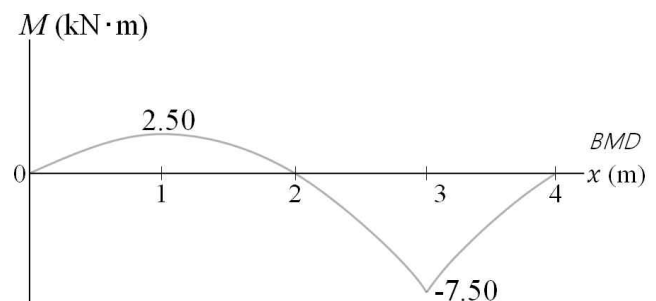
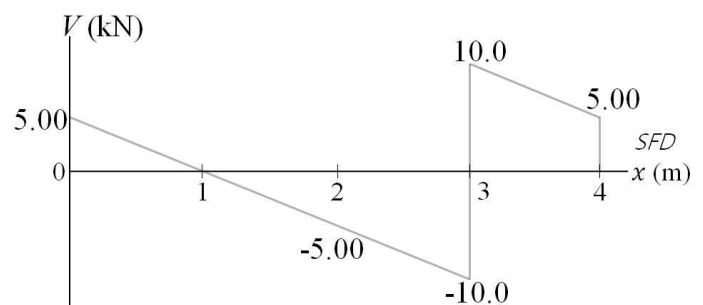
$$V = 5 - 5x + 20$$

$$M = 5x - 5x \cdot \frac{x}{2} + 20(x - 3)$$

$$\text{ANS : } V = -5x + 25$$

$$\text{ANS : } M = -\frac{5}{2}x^2 + 25x - 60$$

- 3) Draw the shear and moment diagram of the beam. Indicate the absolute maximum shear force (V_{\max}) and the maximum bending moment (M_{\max}) at the diagram.



3. The composite beam shown in Fig. is supported by a roller at A, by a hinge at B and by fixed support at C.

1) Determine the reaction force A_y (kN) at point A and C_x (kN), C_y (kN), M_c (kN-m) at point C.

Support Reactions: For member AB

$$\hookrightarrow + \sum M_B = 0; \quad 50(4/3) - A_y(4) = 0 \quad A_y = 16.67 \text{ kN}$$

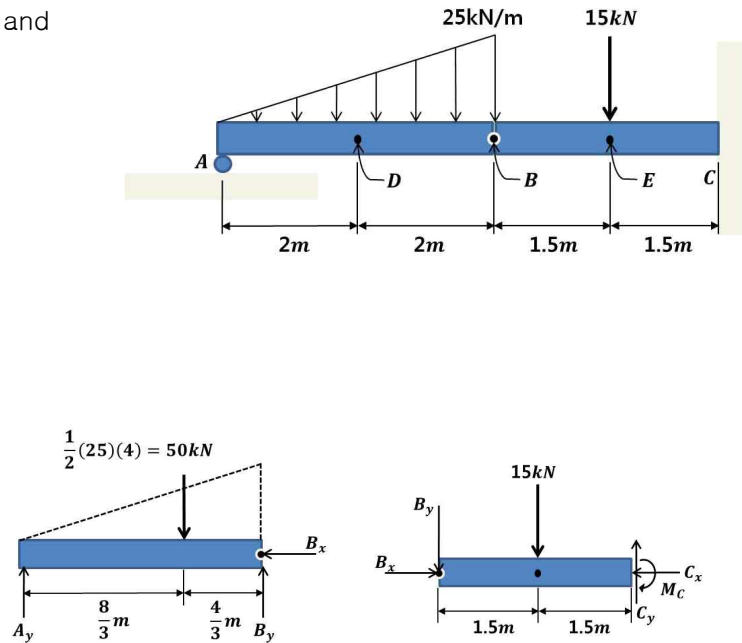
$$\xrightarrow{+} \sum F_x = 0; \quad B_x = 0$$

$$+ \uparrow \sum F_y = 0; \quad B_y + 16.67 - 50 = 0 \quad B_y = 33.33 \text{ kN}$$

$$\sum F_x = 0, \quad C_x = 0$$

$$\sum F_y = 0, \quad C_y = 15 + 33.3 = 48.3 \text{ kN}$$

$$\sum M_c = 0, \quad M_c = 15(1.5) + 33.3(3) = 122.5 \text{ kN} \cdot \text{m}$$



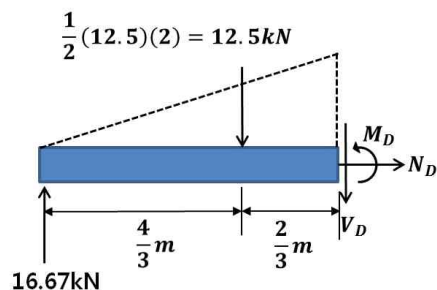
ANS : $A_y = 16.7 \text{ kN}$, $C_x = 0 \text{ kN}$, $C_y = 48.3 \text{ kN}$, $M_c = 122.5 \text{ kN-m}$

2) Determine the resultant internal loadings in the beam N_D (kN), V_D (kN) and M_D (kN-m) at the cross section through the point D.

$$\xrightarrow{+} \sum F_x = 0; \quad N_D = 0$$

$$+ \uparrow \sum F_y = 0; \quad 16.67 - 12.5 - V_D = 0 \\ V_D = 4.17 \text{ kN}$$

$$\hookrightarrow + \sum M_D = 0; \quad M_D + 12.5\left(\frac{2}{3}\right) - 16.67(2) = 0 \\ M_D = 25.17 \text{ kN} \cdot \text{m}$$



ANS : $N_D = 0 \text{ kN}$, $V_D = 4.2 \text{ kN}$, $M_D = 25.2 \text{ kN-m}$

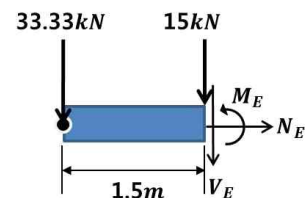
3) Determine the resultant internal loadings in the beam N_E (kN), V_E (kN) and M_E (kN-m) at the cross section through the point E. Point E is just to the right of the 15 kN load.

Equations of Equilibrium: For point E

$$\xrightarrow{+} \sum F_x = 0; \quad N_E = 0$$

$$+ \uparrow \sum F_y = 0; \quad -33.33 - 15 - V_E = 0 \quad V_E = -48.33 \text{ kN}$$

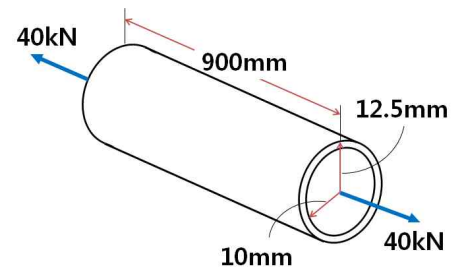
$$\hookrightarrow + \sum M_E = 0; \quad M_E + 33.33(1.5) = 0 \quad M_E = -50.00 \text{ kN} \cdot \text{m}$$



ANS : $N_E = 0 \text{ kN}$, $V_E = 48.3 \text{ kN}$, $M_E = 50 \text{ kN-m}$

4. The thin-walled tube is subjected to an axial force of 40 kN. If the tube elongates 3 mm and its circumference decreases 0.09 mm. The tube's material behaves elastically.

- 1) Determine the normal stress σ_a (MPa) for an axial direction.
- 2) Determine the normal strain ϵ_a (10^{-3} mm/mm) for an axial direction.
- 3) Determine the modulus of elasticity E (GPa) of the material.
- 4) Determine the Poisson's ratio ν of the material.
- 5) Determine the shear modulus G (GPa) of the material.



$$(1) \sigma = \frac{P}{A} = \frac{40 \times 1000}{\pi(0.0125^2 - 0.01^2)} = 226.35 \text{ MPa}$$

$$(2) \epsilon_a = \frac{\delta}{L} = \frac{3}{900} = 3.3333 \times 10^{-3} \text{ mm/mm}$$

$$(3) E = \frac{\sigma}{\epsilon_a} = \frac{226.35}{3.3333 \times 10^{-3}} = 67905 \text{ MPa} = 67.9 \text{ GPa}$$

$$(4) r = 2\pi \times 12.5 - 0.09 = 78.4498 \text{ mm}$$

$$\frac{78.4498}{2\pi} = 12.4857 \text{ mm}$$

$$\epsilon_{lat} = \frac{r - r_0}{r_0} = \frac{12.4857 - 12.5}{12.5} = -1.144 \times 10^{-3}$$

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_a} = -\frac{-1.144 \times 10^{-3}}{3.3333 \times 10^{-3}} = 0.3432$$

$$(5) G = \frac{E}{2(1 + \nu)} = \frac{67.91 \times 10^9}{2(1 + 0.3432)} = 25.28 \times 10^9 = 25.3 \text{ GPa}$$

ANS : 1) $\sigma_a = 226 \text{ MPa}$, 2) $\epsilon_a = 3.33 (10^{-3}) \text{ mm/mm}$, 3) $E = 67.9 \text{ GPa}$, 4) $\nu = 0.344$, 5) $G = 25.3 \text{ GPa}$

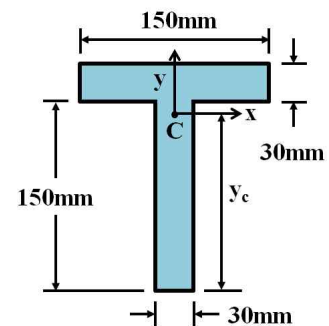
5.

- 1) Determine the centroid C from the bottom y_c (mm) of the cross-sectional area for the T-beam shown in Fig.

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{[75 \text{ mm}](150 \text{ mm})(30 \text{ mm}) + [165 \text{ mm}](30 \text{ mm})(150 \text{ mm})}{(150 \text{ mm})(30 \text{ mm}) + (30 \text{ mm})(150 \text{ mm})}$$

$$= 120 \text{ mm}$$

ANS: $y_c = 120 \text{ mm}$



- 2) Determine the moment of inertia I_x (10^6 mm^4) of the cross-sectional area of the T-beam shown in Fig. about the centroidal x axis.

$$I = \sum (\tilde{I}_x + A d_y^2)$$

$$= \left[\frac{1}{12} (30 \text{ mm})(150 \text{ mm})^3 + (30 \text{ mm})(150 \text{ mm})(120 \text{ mm} - 75 \text{ mm})^2 \right]$$

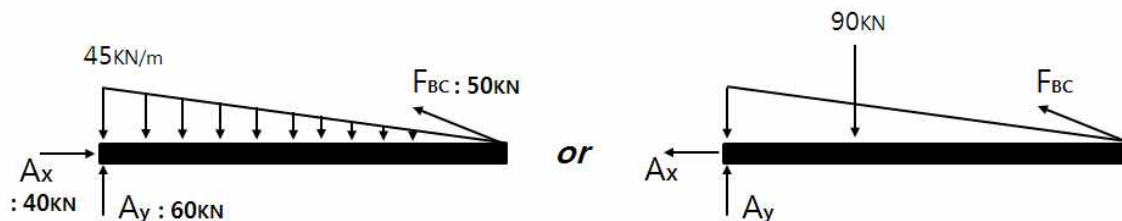
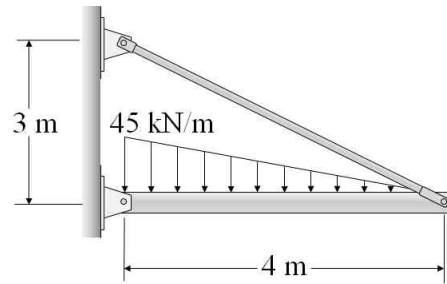
$$+ \left[\frac{1}{12} (150 \text{ mm})(30 \text{ mm})^3 + (150 \text{ mm})(30 \text{ mm})(60 \text{ mm} - 15 \text{ mm})^2 \right]$$

$$= 27.0 (10^6) \text{ mm}^4$$

ANS: $I_x = 27 (10^6) \text{ mm}^4$

6. The rigid bar is supported by the pin connected rod CB that has a cross-sectional area of 500mm^2 and the Young's modulus of the rod CB is 200GPa .

1) Draw the free body diagram of the rigid bar AB and determine the reaction force $A_x(\text{kN})$ and $A_y(\text{kN})$ at point A and the force acted in the rod CB $F_{BC}(\text{kN})$.



ANS : $A_x = 40 \text{ kN}$, $A_y = 60 \text{ kN}$, $F_{BC} = 50 \text{ kN}$

- 2) Determine the average normal stress of the rod CB σ_{CB} (MPa).

$$F_{BC} \cdot \frac{3}{5} \times 4 - 90 \times \frac{4}{3} = 0, \therefore F_{BC} = 50 \text{ kN} = 50,000 \text{ N}$$

$$\sigma = \frac{N}{A} = \frac{P}{A} = \frac{50,000 \text{ N}}{500 \text{ mm}^2} = 100 \text{ N/mm}^2 = 100 \text{ MPa} = 0.1 \text{ GPa} = 10^5 \text{ kPa} = 10^8 \text{ Pa}$$

ANS : $\sigma_{CB} = 100 \text{ MPa}$

- 3) Determine the average shear stress in the 20mm-diameter pin at A τ_A (MPa) which is subjected to double shear.

$$\tau = \frac{V}{A} = \frac{\sqrt{40^2 + 60^2}}{2A} = \frac{72,110 \text{ N}}{200\pi \text{ mm}^2} = \frac{721.1}{2\pi} \text{ MPa} = 115 \text{ MPa}$$

ANS : $\tau_A = 115 \text{ MPa}$

- 4) Determine the relative displacement between two points C and B on the rod δ_{BC} (mm).

$$\delta_{BC} = \frac{PL}{AE} = \frac{50,000 \text{ N} \cdot 5,000 \text{ mm}}{500 \text{ mm}^2 \cdot 200,000 \text{ MPa}} = 2.5 \text{ mm} \quad \text{or} \quad \frac{50 \times 10^3 \times 5}{0.5 \times 10^{-3} \times 200 \times 10^9} = 2.5 \times 10^{-3} \text{ m} = 2.5 \text{ mm}$$

ANS : $\delta_{BC} = 2.5 \text{ mm}$

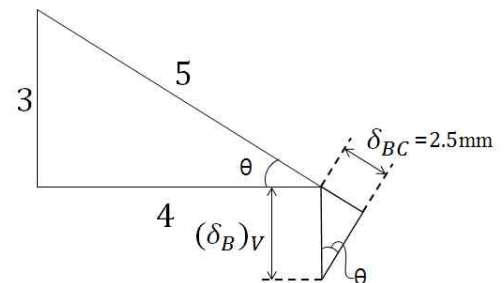
- 5) Determine the vertical displacement of the bar at B $(\delta_B)_V$ (mm).

$$\delta_{BB'} = 2.5 \times \frac{5}{3} = 4.17 \text{ mm}$$

or

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

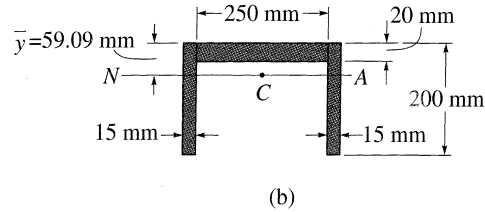
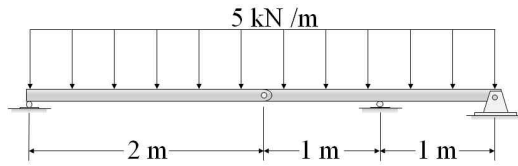
$$\delta_{BB'} = \frac{\delta_{BC}}{\sin \theta} = \frac{2.5 \times 10^{-3}}{\sin 36.87^\circ} = 4.167 \times 10^{-3} \text{ m} = 4.17 \text{ mm}$$



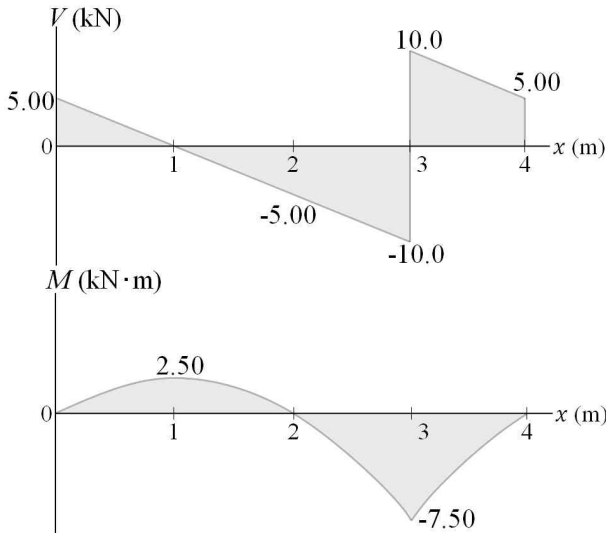
ANS : $(\delta_B)_V = 4.17 \text{ mm}$

7. A beam ($E=200\text{GPa}$, Cross-section is as shown.) is subjected to the load as shown.

- 1) Draw the shear and moment diagram of the beam. Indicate the absolute maximum shear force (V_{\max}) and the maximum bending moment (M_{\max}) at the diagram.



ANS:



- 2) The cross section of the beam is shown as fig. Determine the moment of inertia (I) and the maximum first moment (Q_{\max}) of the cross-sectional area about the centroidal N-A axis.

Section Properties :

$$I = \left[\frac{1}{12} (250)(20)^3 + (250)(20)(59.09 - 10)^2 \right] + 2 \left[\frac{1}{12} (15)(200)^3 + (15)(200)(100 - 59.09)^2 \right]$$

$$= 42.26 (10^6) \text{ mm}^4$$

$$Q_{\max} = \bar{y}' A' = 49.09 (250)(20) + 2 (59.09/2)(59.09)(15)$$

$$= 297.8 (10^3) \text{ mm}^3$$

OR

$$Q_{\max} = \bar{y}' A' = (140.9/2)(30)(140.9) = 297.8 (10^3) \text{ mm}^3$$

ANS: $I = 42.26 (10^6) \text{ mm}^4$, $Q_{\max} = 297.8 (10^3) \text{ mm}^3$

- 3) Determine the maximum bending stress (σ_{\max}) and the maximum shear stress (τ_{\max}) in the beam.

$$\sigma_{\max} = -\frac{Mc}{I} = -\frac{(-7.50 (10^6) \text{ N} \cdot \text{mm})(-140.9 \text{ mm})}{42.26 (10^6) \text{ mm}^4} = -25.01 \text{ MPa}$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{10 (10^3) \text{ N} (297.8) (10^3) \text{ mm}^3}{42.26 (10^6) \text{ mm}^4 (30) \text{ mm}} = 2.35 \text{ MPa}$$

ANS: $\sigma_{\max} = -25.0 \text{ MPa}$, $\tau_{\max} = 2.3 \text{ MPa}$

8. A shaft delivers a maximum power of 28260kW and causes the shaft to rotate at 1200 rpm. The outer diameter of the shaft is 340mm and Inner diameter of the shaft is 260mm.

- 1) Determine the internal torque (T) on the shaft.

$$P = \frac{2\pi n T}{60 \times 1000} = \frac{2\pi n T}{60000}, \quad 28260 \text{ kW} = \frac{2\pi (1200 \text{ rpm}) T}{60000}$$

$$T = 225 \text{ kN-m} = 225 (10^6) \text{ N-mm}$$

ANS: $T = 225 \text{ kN-m}$

- 2) Determine the maximum shear stress (τ_{\max}) developed in the shaft.

$$\tau_{\max} = \frac{Tc}{J} = \frac{225 (10^6) (170)}{\frac{\pi}{2} (170^4 - 130^4)} = 44.3 \text{ MPa}$$

ANS: $\tau_{\max} = 44.3 \text{ MPa}$

- 3) Determine the angle of twist (Φ , rad) of the shaft when the length of the shaft is 60m. ($G=75 \text{ GPa}$)

$$\phi = \frac{TL}{GJ} = \frac{225 (10^6) (60 (10^3))}{75 (10^3) \frac{\pi}{2} (170^4 - 130^4)} = 0.2086 \text{ rad} = 11.9^\circ = 12^\circ$$

ANS: $\Phi = 0.21 \text{ rad}$

9. A shaft delivers a maximum power of 2000kW and causes the shaft to rotate at 1200 rpm.
The outer diameter of the shaft is 200mm and Inner diameter of the shaft is 180mm.

1) Determine the internal torque (T) on the shaft.

$$P = \frac{2\pi n T}{60 \times 1000} = \frac{2\pi n T}{60000}, \quad 2000 \text{ kW} = \frac{2\pi(1200 \text{ rpm}) T}{60000},$$

$$T = 15.92 \text{ kN-m} = 15.92(10^6) \text{ N-mm}$$

ANS: $T = 15.9 \text{ kN-m}$

2) Determine the maximum shear stress (τ_{\max}) developed in the shaft.

$$\tau_{\max} = \frac{Tc}{J} = \frac{15.92(10^6) 100}{\frac{\pi}{2}(100^4 - 90^4)} = 29.47 \text{ MPa}$$

ANS: $\tau_{\max} = 29.5 \text{ MPa}$

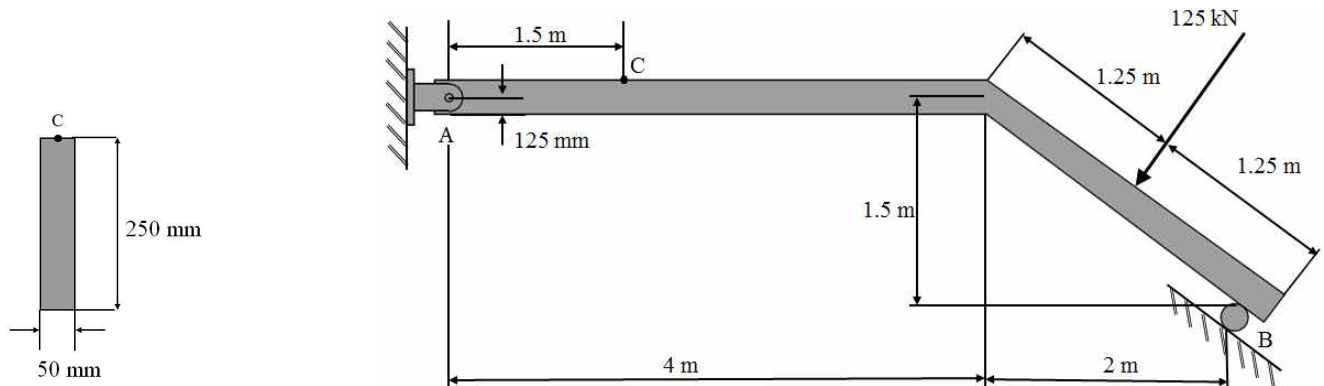
3) Determine the angle of twist (Φ) of the shaft when the length of the shaft is 20m. ($G=75 \text{ GPa}$)

$$\phi = \frac{TL}{GJ} = \frac{15.92(10^6) 20(10^3)}{75(10^3) \frac{\pi}{2}(100^4 - 90^4)} = 0.0786 \text{ rad} = 4.5^\circ$$

ANS: $\Phi = 0.0786 \text{ rad}$

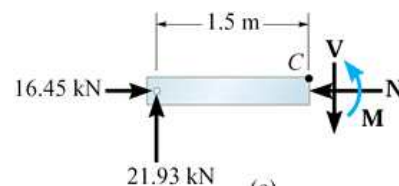
10. The member shown in Fig. has a rectangular cross section.

1) Determine the resultant internal loadings (N, V, M) that products at the section C.



$$A_x - \frac{3}{5} \times 125 + \frac{3}{5} \times 97.59 = 0 \quad \therefore A_x = 16.45$$

$$A_y - \frac{4}{5} \times 125 + \frac{4}{5} \times 97.59 = 0 \quad \therefore A_y = 21.93$$



ANS: $N = -16.45 \text{ kN}, \quad V = 21.93 \text{ kN}, \quad M = 32.89 \text{ kN-m}$

2) Determine the state of stress (σ_c, τ_c) that the loading products at point C.

Normal Force

$$\sigma_c = \frac{N}{A} = \frac{-16.45 \text{ kN}}{(0.050 \text{ m})(0.250 \text{ m})} = -1.32 \text{ MPa}$$

Shear Force

$\tau_c = 0$ since point C is located at the top of the member.

Bending Moment

$$\sigma_c = -\frac{Mc}{I} = -\frac{(32.89 \text{ kN} \cdot \text{m})(0.125 \text{ m})}{\left[\frac{1}{12}(0.050 \text{ m})(0.250 \text{ m})^3\right]} = -63.15 \text{ MPa}$$

Superposition.

$$\sigma_c = 1.32 \text{ MPa} + 63.15 \text{ MPa} = 64.5 \text{ MPa}$$

ANS: $\sigma_c = -64.5 \text{ MPa}, \quad \tau_c = 0 \text{ MPa}$

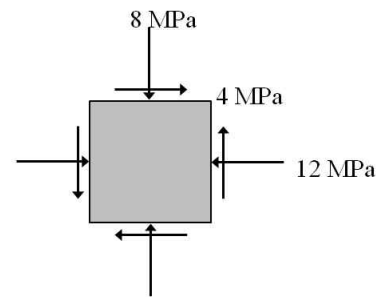
11. The state of plane stress at a point is represented by the element as shown.

- 1) Determine the state of stress ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$) at the point on another element oriented 30° clockwise from the position shown.

$$\begin{aligned}\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-12 - 8}{2} + \frac{-12 + 8}{2} \cos 2(-30^\circ) + 4 \sin 2(-30^\circ) \\ &= -14.46 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_{y'} &= \frac{-12 - 8}{2} - \frac{-12 + 8}{2} \cos 2(-30^\circ) - 4 \sin 2(-30^\circ) \\ &= -5.54 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{-12 + 8}{2} \sin 2(-30^\circ) + 4 \cos 2(-30^\circ) \\ &= 0.27 \text{ MPa}\end{aligned}$$



ANS: $\sigma_{x'} = -14.46 \text{ MPa}$, $\sigma_{y'} = -5.54 \text{ MPa}$, $\tau_{x'y'} = 0.27 \text{ MPa}$

- 2) Determine the principal stresses (σ_1 , σ_2) and the orientation (θ_{p1} , θ_{p2}) of the element at the point.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-12 - 8}{2} = -10 \text{ MPa}, \quad R = \sqrt{2^2 + 4^2} = 4.47$$

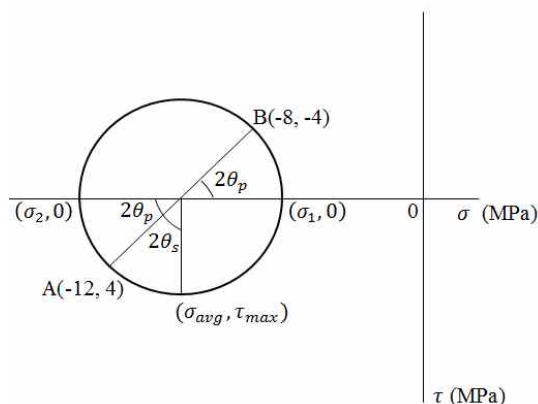
$$\sigma_1 = \sigma_{avg} + R = -10 + 4.47 = -5.53 \text{ MPa}$$

$$\sigma_2 = \sigma_{avg} - R = -10 - 4.47 = -14.47 \text{ MPa}$$

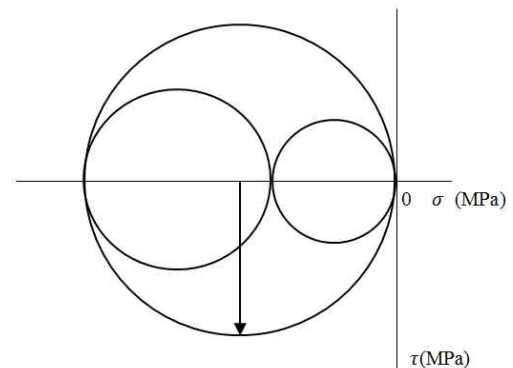
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(4)}{-12 + 8} = -2, \quad 2\theta_p = -63.4^\circ, \quad \theta_{p2} = -31.7^\circ, \quad \theta_{p1} = -31.7^\circ + 90^\circ = 58.3^\circ$$

ANS: $\sigma_1 = -5.53 \text{ MPa}$, $\sigma_2 = -14.47 \text{ MPa}$, $\theta_{p1} = 58.3^\circ$, $\theta_{p2} = -31.7^\circ$

- 3) Draw Mohr's circle for the state of stress at the point and determine the absolute maximum shear stress ($\tau_{\max-\text{abs}}$) developed at the point.



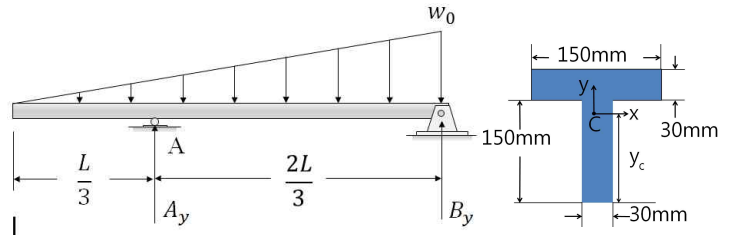
OR



$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{0 - (-14.47)}{2} = 7.24 \text{ MPa}$$

ANS: $\tau_{\max-\text{abs}} = 7.24 \text{ MPa}$

12. A beam ($E=200\text{GPa}$, Cross-section is as shown.) is subjected to the load as shown. Where $w_0=3.6\text{kN/m}$, $L=9\text{m}$.



- 1) Draw the shear and moment diagram of the beam. Indicate the absolute maximum shear force (V_{\max}) and the maximum bending moment (M_{\max}) at the diagram.

ANS:

$$A_y = B_y = \frac{w_0 L}{4} = 8.1\text{kN}$$

$$V_{\max} = -B_y = -\frac{w_0 L}{4} = -8.1\text{kN}$$

$$V = \frac{w_0 L}{4} - \frac{w_0 x^2}{2L} = 0 \quad \therefore x = 0.707L$$

$$M_{\max} = \frac{w_0 L}{4} \left(0.707L - \frac{L}{3}\right) - \frac{w_0}{6L} (0.707L)^3 = 0.0345w_0 L^2 = 10.06\text{kNm}$$

- 2) The cross section of the beam is shown as fig. Determine the moment of inertia (I_x) and the maximum first moment (Q_{\max}) of the cross-sectional area about the centroidal x-axis.

Section Properties :

$$y_c = \frac{\sum \tilde{y}A}{\sum A} = \frac{[75\text{mm}](150\text{mm})(30\text{mm}) + [165\text{mm}](30\text{mm})(150\text{mm})}{(150\text{mm})(30\text{mm}) + (30\text{mm})(150\text{mm})} = 120\text{mm}$$

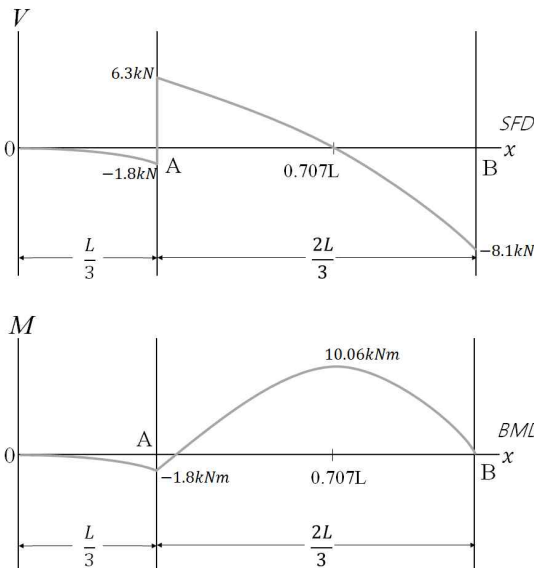
$$I_x = \sum (\bar{I}_x + Ad_y^2) = \left[\frac{1}{12} (30\text{mm})(150\text{mm})^3 + (30\text{mm})(150\text{mm})(120\text{mm} - 75\text{mm})^2 \right] + \left[\frac{1}{12} (150\text{mm})(30\text{mm})^3 + (150\text{mm})(30\text{mm})(60\text{mm} - 15\text{mm})^2 \right] = 27.0(10^6)\text{mm}^4$$

$$Q_{\max} = \bar{y}' A' = 45(150)(30) + 15(30)(30) = 216(10^3)\text{mm}^3$$

OR

$$Q_{\max} = \bar{y}' A' = 60(30)(120) = 216(10^3)\text{mm}^3$$

$$\text{ANS: } I_x = 27.0(10^6)\text{mm}^4, \quad Q_{\max} = 216(10^3)\text{mm}^3$$



- 3) Determine the maximum bending stress (σ_{\max}) and the maximum shear stress (τ_{\max}) in the beam.

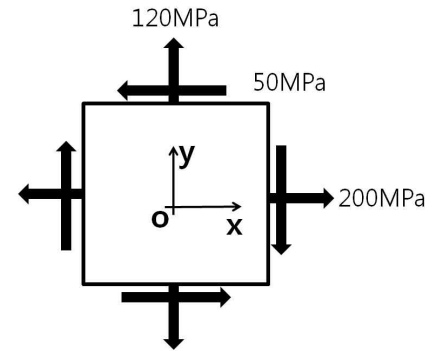
$$\sigma_{\max} = -\frac{Mc}{I} = -\frac{(10.06(10^6)\text{N} \cdot \text{mm})(-120\text{mm})}{27(10^6)\text{mm}^4} = 44.7\text{MPa}$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = -\frac{8.1(10^3)\text{N}(216)(10^3)\text{mm}^3}{27(10^6)\text{mm}^4(30\text{mm})} = -2.16\text{MPa}$$

$$\text{ANS: } \sigma_{\max} = 44.7\text{MPa}, \quad \tau_{\max} = -2.2\text{MPa}$$

13. The state of plane stress at a point is represented by the element as shown.

- 1) Determine the state of stress($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$) at the point on another element oriented 45° clockwise from the position shown.



$$\begin{aligned}\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{200 + 120}{2} + \frac{200 - 120}{2} \cos 2(-45^\circ) - 50 \sin 2(-45^\circ) \\ &= 210 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_{y'} &= \frac{200 + 120}{2} - \frac{200 - 120}{2} \cos 2(-45^\circ) + 50 \sin 2(-45^\circ) \\ &= 110 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{200 - 120}{2} \sin 2(-45^\circ) - 50 \cos 2(-45^\circ) \\ &= 40 \text{ MPa}\end{aligned}$$

ANS: $\sigma_{x'} = 210 \text{ MPa}$, $\sigma_{y'} = 110 \text{ MPa}$, $\tau_{x'y'} = 40 \text{ MPa}$

- 2) Determine the principal stresses(σ_1 , σ_2) and the orientation (θ_{p1} , θ_{p2}) of the element at the point.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{200 + 120}{2} = 160 \text{ MPa}, \quad R = \sqrt{40^2 + 50^2} = 64.03$$

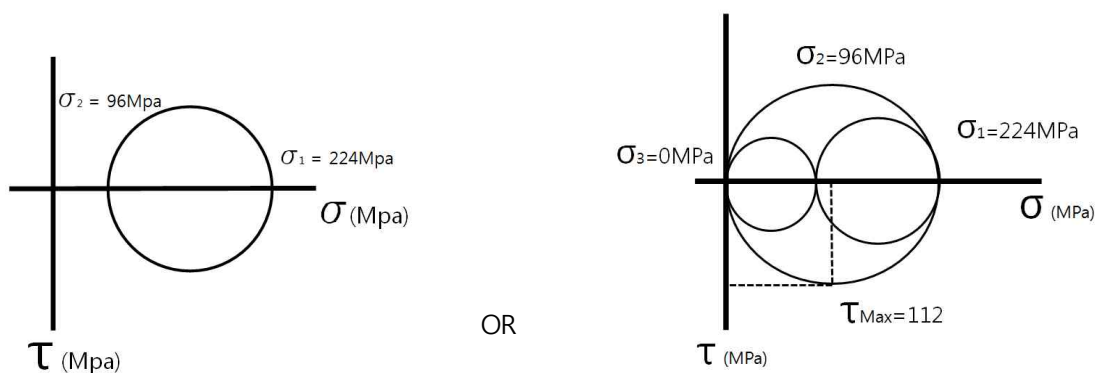
$$\sigma_1 = \sigma_{avg} + R = 160 + 64.03 = 224.03 \text{ MPa}$$

$$\sigma_2 = \sigma_{avg} - R = 160 - 64.03 = 95.97 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-2(50)}{200 - 120} = -1.25, \quad 2\theta_p = -51.34^\circ, \quad \theta_{p1} = -25.67^\circ, \quad \theta_{p2} = -25.67^\circ + 90^\circ = 64.33^\circ$$

ANS: $\sigma_1 = 224 \text{ MPa}$, $\sigma_2 = 96 \text{ MPa}$, $\theta_{p1} = -25.7^\circ$, $\theta_{p2} = 64.3^\circ$

- 3) Draw Mohr's circle for the state of stress at the point and determine the absolute maximum shear stress($\tau_{\max\text{-abs}}$) developed at the point.



$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{224 - 0}{2} = 112 \text{ MPa}$$

ANS: $\tau_{\max\text{-abs}} = 112 \text{ MPa}$